Blocks of Finite Groups and Beyond

Christine Bessenrodt (Hannover):

Restricted Kronecker products for the symmetric groups

Already many years ago, James and Kerber stated in their famous book: 'The inner tensor product of two ordinary irreducible representations of S_n is in general reducible, so the question arises how to evaluate its decomposition into irreducible constituents.' As in those days, still 'no satisfactory answer to this question is known'. Indeed, apart from trivial cases, such Kronecker products are always reducible, and it is a notoriously difficult problem to determine the corresponding Kronecker coefficients; positive combinatorial formulae are known so far only for very special partition labels. In the talk, the focus will be on classification problems for Kronecker products with restricted Kronecker coefficients and related problems, and some recent progress will be discussed.

Serge Bouc (Amiens):

Representations of finite sets

In this joint work (in progress) with Jacques Thévenaz, we develop the representation theory of finite sets and correspondences: for a commutative ring k, we consider correspondence functors, i. e. k-linear functors from the category of finite sets, where morphisms are k-linear combinations of correspondences, to the category of k-modules. We discuss various properties of the abelian category F_k of correspondence functors: finite generation and finite length, bounded generation, self-injectivity and symmetry, global dimension, projective, injective, and simple objects, tensor structure, etc.

We show in particular that the simple correspondence functors over k are parametrized by pairs consisting of a finite poset and a simple k-linear representation of its automorphism group. The case of a total order is of special interest, as the associated simple functor is also projective and injective (when k is a field). We also attach a correspondence functor F_T to any finite lattice T, and show that F_T is projective if and only if T is distributive. This construction extends to a fully faithful k-linear functor from a suitable category kL of finite lattices to F_k.

Charles Eaton (Manchester):

Donovan's conjecture

I will give a brief overview on Donovan's conjecture and related problems concerning the Morita equivalence classes of blocks of finite groups with a given defect group, before concentrating on the case of abelian defect groups for the prime two.

Karin Erdmann (Oxford):

Some representations of quaternion algebras

Let A be a 2-block with quaternion defect groups, or more generally an algebra of quaternion type. We classify τ-rigid modules for such algebras. We also give some examples of 3-cluster tilting modules.

Paul Fong (Chicago):

The Alperin weight conjecture for S_n and $\mathbf{GL}(n, q)$ revisited

A direct connection between the number of simple modules and the weights in these groups is given in terms of a simple algebraic encoding.

Eugenio Giannelli (Kaiserslautern):

Signed Young permutation modules of the symmetric group

Given λ a partition of n we denote by S_λ the corresponding Young subgroup of S_n. The p-permutation modules obtained by inducing any one-dimensional representation of S_λ to the full symmetric group S_n are called signed Young permutation modules. In this talk, I will start by describing the main properties of signed Young permutation modules and by explaining the important role played by this family of modules in the representation theory of symmetric groups.

In the second part of the talk I will present some new results on the modular structure of these modules. In particular, I will describe some reductions for signed p-Kostka numbers, namely the multiplicities of indecomposable signed Young modules as direct summands of signed Young permutation modules. I will conclude by giving a complete classification of the indecomposable signed Young permutation modules. This is joint work with Kay Jin Lim and Mark Wildon.

László Héthelyi (Budapest):

On p-stability in fusion systems

This is joint work with M. Szőke and A. Zalesskii. Let $p \neq 2$ be a prime number. p-stability for groups was originally defined by D. Gorenstein and J. H. Walter in the early 60's. The definition has been slightly changed by Glauberman in 1968 and 1971. The group $\mathrm{Qd}(p)$ is defined as the semidirect product of the special linear group $\mathrm{SL}_2(p)$ with an elementary abelian group V of order p^2, where $\mathrm{SL}_2(p)$ acts on V via its natural action. It was shown by Glauberman that all sections of a finite group are p-stable if and only if it does not involve $\mathrm{Qd}(p)$. $\mathrm{Qd}(p)$-free fusion systems were defined and investigated by R. Kessar and M. Linckelmann in 2008.

We define p-stability for fusion systems and section p-stability. We show that the fusion system is $\mathrm{Qd}(p)$-free if and only if it is section p-stable. It is easily seen that section p-stable fusion systems are soluble and hence they come from groups. We prove that a fusion system is section p-stable if and only if the underlying group is p-stable. We slightly refine the theorem of Glauberman by showing that a group is section p-stable if and only if there is no p-centric p-subgroup whose normaliser involves $\mathrm{Qd}(p)$. We also investigate the question when a finite simple group gives a soluble fusion system and what the connection is between having a strongly closed abelian subgroup and being section p-stable.

Radha Kessar (London):

On anchors of irreducible characters

Let p be a prime number and $(K, \mathcal{O}, k)$ a p-modular system. To each ordinary (K-valued) irreducible character of a finite group G can be associated in a natural way a unique conjugacy class of p-subgroups of G, which we call the p-anchors of the character. We will present some connections between anchors with other p-groups which can be attached to irreducible characters, such as defect groups and vertices in the sense of Green. This is joint work with Burkhard Külshammer and Markus Linckelmann.

Shigeo Koshitani (Chiba):

Endo-trivial modules in the dihedral Sylow 2-subgroups case

Endo-trivial modules in tame representation types have been treated by Carlson-Mazza-Thévenaz in their paper of 2013, except for the case that the finite groups we are looking at have dihedral Sylow 2-subgroups (including Klein-four groups). In this talk we will be discussing the remaining case, namely, dihedral Sylow 2-subgroups case, hopefully our result makes the tame representation type case finished. This is joint work with Caroline Lassueur and a continuation of our previous paper [Koshitani-Lassueur: 'Endo-trivial modules for finite groups with Klein-four Sylow 2-subgroups', to appear in Manuscripta Math.].

Markus Linckelmann (London):

On cohomological Mackey functors

It was shown by Rognerud that a source algebra equivalence between two block algebras induces an equivalence between the corresponding categories of cohomological Mackey functors, and that a splendid derived equivalence between blocks induces an equivalence between the derived categories of cohomological Mackey functors. Starting from a version of Yoshida's theorem at the source algebra level, we give alternative proofs of Rognerud's results by explicitly constructing a two-sided tilting complex for the derived categories of cohomological Mackey functors. We use this source algebra version to show further that the subcategories of cohomological Mackey functors which vanish at the trivial group are invariant under splendid stable equivalences. We note finally that some of these results can be expressed in terms of a recollement.

Gunter Malle (Kaiserslautern):

On blocks containing one simple module

We report on ongoing work on a conjecture made by Radha Kessar on blocks of finite groups with only one simple module in positive characteristic. This is joint work with G. Navarro and B. Späth.

John Murray (Maynooth):

Symmetric and symplectic bilinear forms on simple and projective indecomposable modules

Let k be a field and let G be a finite group. We say that a kG-module M has symmetric/symplectic type if M affords a non-degenerate G-invariant symmetric/symplectic bilinear form B. In that case $\mathrm{End}_k(M)$ is an involutary G-algebra; the adjoint of B is an involution σ on $\mathrm{End}_k(M)$ which commutes with the G-action. The setting of involutary G-algebras provides effective tools for developing Clifford and vertex theory for modules with a symmetric or symplectic bilinear form. We can describe orthogonal decompositions, induction from subgroups and relative projectivity of symmetric G-forms.

Given $(\mathrm{End}_k(M), \sigma)$ and $H \leq G$, the involution σ acts on $\mathrm{End}_{kH}(M)$ and on its maximal ideals and points. Given a σ-fixed point, the associated multiplicity algebra is an involutary $N_G(H)$-algebra. Frequently this endows the associated multiplicity module with an equivariant symmetric form. If I is a σ-fixed two sided ideal of $\mathrm{End}_{kH}(M)$, then σ-invariant idempotents of $\mathrm{End}_{kH}(M)/I$ can be lifted to σ-invariant idempotents of $\mathrm{End}_{kH}(M)$. Our focus is on the case that k is an algebraically closed field of characteristic 2. We illustrate with numerous examples from classical and solvable finite groups.

Gabriel Navarro (Valencia):

Real finite groups

We study a conjecture by R. Gow. This is joint work with P. Tiep.

Geoffrey Robinson (Bristol):

A conjectural upper bound for the number of simple modules in a block

This talk is a report on ongoing joint work with G. Malle. We present a new conjectured upper bound for $\ell(B)$, the number of simple modules in a p-block B with defect group D. This is somewhat analogous to, and partly motivated by, Brauer's question on whether we always have $k(B) \leq |D|$. We will outline proofs that the conjecture holds for a large class of blocks, including all p-blocks of p-solvable groups and symmetric and alternating groups, quasisimple Lie type characteristic p groups, sporadic (quasi)simple groups. If time permits we will discuss the situation for Lie type quasisimple groups in characteristic different from p, and some of the Clifford theoretic issues likely to be involved in any proof.

Benjamin Sambale (Jena):

Cartan matrices and Brauer's $k(B)$-conjecture

It is known that the order of a defect group D of a block B of a finite group is the unique largest elementary divisor of the Cartan matrix C of B. On the other hand, C factorizes as $C = Q^{\mathrm{T}}Q$ where Q is the decomposition matrix of B with $k(B)$ (non-zero) rows. In my talk I will show how this connection can be used in order to attack Brauer's $k(B)$-Conjecture which asserts $k(B) \leq |D|$.

Leonard Scott (Charlottesville):

Using algebraic groups to go 'beyond'

I show how algebriac group representation theory and cohomology have led, through work of various authors, to a counterexample to a 1961 maximal subgroups conjecture true for all solvable and possibly all simple groups. On the positive side, similar representation theory pursuits have led to extremely strong positive answers to the Donovan conjecture in the case of finite groups of Lie type in the defining characteristic, and bounds for the cohomology with irreducible coefficients for such groups in any characteristic, depending only on the Lie rank and homological degree.

Bhama Srinivasan (Chicago):

Blocks of GL(n, q), old and new

The ℓ-blocks of $\mathrm{GL}(n, q)$, where ℓ is a prime not dividing q, were classified by Fong-Srinivasan in 1982, and were analogous to blocks of symmetric groups with a combinatorial description. More recently, the work of Lascoux-Leclerc-Thibon and Chuang-Rouquier gives a description of the blocks in terms of the action of a special linear group on a vector space indexed by partitions. Finally Lusztig functors enter the picture. These different ways of looking at blocks will be described.

Jacques Thévenaz (Lausanne):

It's a long way to the land of endo-trivial modules

Endo-trivial modules were first defined independently by Alperin and Dade in 1978. Dade classified them for abelian p-groups and introduced on this occasion some fundamental new ideas in modular representation theory. A quater of a century later, endo-trivial modules were classified in the case of p-groups, using heavy cohomological machinery. Now ten more years have passed and some new techniques emerged, which shed some new light on the question. This talk will give an introduction to the subject and a survey of its main achievements. It will also present some recent results giving a classification for several classes of finite groups.

Tomoyuki Wada (Tokyo):

Number of irreducible Brauer characters of height 0 in 2-blocks of finite groups

We show that in most 2-blocks of finite groups, the number of irreducible Brauer characters of height zero is odd. We call such a 2-block good, because the Cartan matrix of the block has a good property for integrality of its eigenvalues. We prove that 2-blocks of tame representation type are good except only one case $\mathrm{SD}(3C)_2$. We finally find an example of a bad 2-block that is a faithful and full-defect 2-block of the triple cover 3.Suz of the sporadic simple Suzuki group.

Wolfgang Willems (Magdeburg):

Indecomposable quasi-projective characters in a block

For an irreducible p-Brauer character φ let Φ_φ denote the ordinary character associated to the projective cover of the module afforded by φ. We call an ordinary character χ resp. a p-Brauer character β quasi-projective if χ resp. β can be written as $\sum_\varphi a_\varphi \Phi_\varphi$ with $a_\varphi \in \mathbb{Z}$. In the talk we investigate indecomposable quasi-projective characters and discuss some open problems.